Exercise Set 9.1

Practice Exercises
In Exercises 1–16, use dimensional analysis to convert the quantity to the indicated unit. If necessary, round the answer to two decimal places.

1. 30 in. to ft
2. 100 in. to ft
3. 30 in. to in.
4. 100 ft to in.
5. 6 in. to yd
6. 21 in. to yd
7. 6 yd to in.
8. 21 yd to in.
9. 6 yd to ft
10. 12 yd to ft
11. 6 ft to yd
12. 12 ft to yd
13. 23,760 ft to mi
14. 19,800 ft to mi
15. 0.75 mi to ft
16. 0.25 mi to ft

In Exercises 17–26, use the diagram in the box on page 579 to convert the given measurement to the unit indicated.

17. 5 in to cm
18. 3 in to cm
19. 16.3 cm to in
20. 0.37 km to m
21. 317 cm to ft
22. 8.64 km to cm
23. 0.023 mm to m
24. 0.00037 km to cm
25. 2196 mm to dm
26. 71 dm to km

In Exercises 27–44, use the following English and metric equivalents, along with dimensional analysis, to convert the given measurement to the unit indicated.

English and Metric Equivalents

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>in.</td>
<td>= 0.0254 m</td>
</tr>
<tr>
<td>ft</td>
<td>= 0.3048 m</td>
</tr>
<tr>
<td>yd</td>
<td>= 0.9144 m</td>
</tr>
<tr>
<td>mi</td>
<td>= 1.6093 km</td>
</tr>
<tr>
<td>cm</td>
<td>= 0.01 m</td>
</tr>
<tr>
<td>m</td>
<td>= 1 m</td>
</tr>
</tbody>
</table>

Use 1 mi = 1.6 km to solve Exercises 45–48.

45. Express 96 kilometers per hour in miles per hour.
46. Express 104 kilometers per hour in miles per hour.
47. Express 45 miles per hour in kilometers per hour.
48. Express 50 miles per hour in kilometers per hour.

 Practice Plus
In Exercises 49–52, use the unit fractions.

49. Convert 3 yd to cm.
50. Convert 8 ft to yd.
51. Convert 762 cm to yd.
52. Convert 1016 cm to yd.

In Exercises 53–54, use the unit fractions.

53. Convert 30 mi to km.
54. Convert 50 mi to km.
55. Use unit fractions to express 120 miles per hour in feet per second.
56. Use unit fractions to express 100 miles per hour in feet per second.

Application Exercises
In Exercises 57–66, selecting from millimeter, meter, and kilometers, determine the best unit of measure to express the given length.

57. A person's height
58. The length of a football field
59. The length of a bus
60. The distance from New York City to Washington, D.C.
61. The distance around a one-acre lot
62. The length of a car
63. The width of a book
64. The altitude of an airplane
65. The diameter of a screw
66. The width of a human foot

In Exercises 67–74, select the best estimate for the measure of the given item.

67. The length of a pen
68. The length of this page
69. The height of a skyscraper
70. The length of a pair of pants
71. The height of a room
72. The length of a rowboat

73. The width of an electric cord
74. The dimensions of a piece of typing paper
75. A baseball diamond measures 27 meters along each side. If a batter scored two home runs in a game, how many kilometers did the batter run?
76. If you jog six times around a track that is 700 meters long, how many kilometers have you covered?
77. The distance from the Earth to the sun is about 93 million miles. What is this distance in kilometers?
78. The distance from New York City to Los Angeles is 4600 kilometers. What is the distance in miles?
79. Exercise 61 gives the approximate length of some of the world's largest rivers. In each exercise, determine which is the larger river and by how many kilometers.
80. Nile: 4350 miles; Amazon: 4600 kilometers
81. Yangtze: 3940 miles; Mississippi: 6250 kilometers

Exercises 81–82 give the approximate height of some of the world's tallest mountains. In each exercise, determine which is the taller mountain and by how many feet. Round to the nearest meter.

81. K2: 8611 meters; Everest: 8849 meters
82. Uluru: 853 meters; Kengtsejungpa: 28170 meters

Exercises 83–84 give the average rainfall of some of the world's wettest places. In each exercise, determine which has the greater average rainfall and by how many inches. Round to the nearest inch.

83. Debundscha (Cameroon): 10280 millimeters; Waialae (Hawaii): 451 inches
84. Monsoon (India): 11870 millimeters; Cherrapunji (India): 498 inches

(Source for Exercises 79–84: Russell Ash, The Top 10 of Everything 2009)

Critical Thinking Exercises
Make Sense? In Exercises 92–95, determine whether each statement makes sense or does not make sense, and explain your reasoning.

92. I can run 4000 meters in approximately one hour.
93. I ran 2000 meters and you ran 2000 yards in the same time, so I ran at a faster rate.
94. The most frequent use of dimensional analysis involves changing units within the metric system.
95. When multiplying by a unit fraction, I put the unit of measure that needs to be introduced in the denominator.

In Exercises 96–100, convert to an appropriate metric unit so that the numerical expression in the given measure does not contain any zeros.

96. 6000 cm
97. 900 m
98. 7000 dm
99. 11,000 mm
100. 0.0002 km

SECTION 9.2 Measuring Area and Volume

9.2 WHAT AM I SUPPOSED TO LEARN?
After you have read this section, you should be able to:

1. Use square units to measure area.
2. Use dimensional analysis to change units for area.
3. Use cubic units to measure volume.
4. Use English and metric units to measure capacity.

A square mile is one way of measuring the area of a state. A state's area is the region within its boundaries. Its population density is its population divided by its area. In this section, we discuss methods for measuring both area and volume.

Writing in Mathematics
Describe the two parts of a measurement.
Describe how to use dimensional analysis to convert 20 inches to feet.
Explain the advantages of the metric system over the English system.
Explain how to change units within the metric system.
You jog 500 meters in a given period of time. The next day, you jog 500 yards over the same time period. On which day was your speed faster? Explain your answer.
What kind of difficulties might arise if the United States immediately eliminated all units of measure in the English system and replaced the system by the metric system?
The United States is the only Westernized country that does not use the metric system as its primary system of measurement. What reasons might be given for continuing to use the English system?
Measuring Area

In order to measure a region that is enclosed by boundaries, we begin by selecting a square unit. A square unit is a square, each of whose sides is one unit in length, illustrated in Figure 9.2. The region in Figure 9.2 is said to have an area of one square unit. The side of the square can be 1 inch, 1 centimeter, 1 meter, 1 foot, or one of any linear unit of measure. The corresponding units of area are the square inch (in.²), the square centimeter (cm²), the square meter (m²), the square foot (ft²), and so on. Figure 9.3 illustrates 1 square inch and 1 square centimeter, drawn to actual size.

EXAMPLE 1 Measuring Area

What is the area of the region shown in Figure 9.4?

SOLUTION

We can determine the area of the region by counting the number of square units contained within the region. There are 12 such units. Therefore, the area of the region is 12 square units.

CHECK POINT 1 What is the area of the region represented by the first two rows in Figure 9.4?

Although there are 12 inches in one foot and 3 feet in one yard, these numerical relationships are not the same for square units.

EXAMPLE 2 Using Square Units to Compute Population Density

After Alaska, Wyoming is the least densely populated state. The population of Wyoming is 568,158 and its area is 97,814 square miles. What is Wyoming’s population density?

SOLUTION

We compute the population density by dividing Wyoming’s population by its area.

\[ \text{population density} = \frac{\text{population}}{\text{area}} = \frac{568,158 \text{ people}}{97,814 \text{ square miles}} \]

Using a calculator and rounding to the nearest tenth, we obtain a population density of 5.8 people per square mile. This means that there is an average of only 5.8 people for each square mile of area.

CHECK POINT 2 The population of California is 39,691,912 and its area is 158,633 square miles. What is California’s population density? Round to the nearest tenth.

EXAMPLE 3 Using Dimensional Analysis on Units of Area

Your author wrote Thinking Mathematically in Point Reyes National Seashore, 40 miles north of San Francisco. The national park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. How large is the national park in square miles?

SOLUTION

We use the fact that 1 square mile = 640 acres to set up our unit fraction:

\[ \frac{1 \text{ mi}^2}{640 \text{ acres}} \]

Thus,

\[ 75,000 \text{ acres} \times \frac{1 \text{ mi}^2}{640 \text{ acres}} = \frac{75,000}{640} \text{ mi}^2 = \frac{75,000}{640} \times \frac{100}{100} \text{ mi}^2 = 117.1875 \text{ mi}^2. \]

The area of Point Reyes National Seashore is approximately 117 square miles.

CHECK POINT 3 The National Park Service administers approximately 84,000,000 acres of national parks. How large is this in square miles?

In Section 9.1, we saw that in most other countries, the system of measurement that is used is the metric system. In the metric system, the square centimeter is used instead of the square inch. The square meter replaces the square foot and the square yard.

The English system uses the acre and the square mile to measure large land areas, where one square mile = 640 acres. The metric system uses the hectare (symbolized ha and pronounced “hectar”). A hectare is about the area of two football fields placed side by side, approximately 2.5 acres. One square mile of land consists of approximately 260 hectares. Just as the hectare replaces the acre, the square kilometer is used instead of the square mile. One square kilometer is approximately 0.38 square mile.
Some basic approximate conversions for units of area are given in Table 9.4.

**TABLE 9.4 Units and Measures, Equivalents for Area**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Equivalent Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 square inch (in²)</td>
<td>6.45 square centimeters (cm²)</td>
</tr>
<tr>
<td>1 square foot (ft²)</td>
<td>0.09 square meter (m²)</td>
</tr>
<tr>
<td>1 square yard (yd²)</td>
<td>0.8 square meter (m²)</td>
</tr>
<tr>
<td>1 square mile (mi²)</td>
<td>2.6 square kilometers (km²)</td>
</tr>
<tr>
<td>1 acre</td>
<td>0.40 hectare (ha)</td>
</tr>
</tbody>
</table>

**EXAMPLE 4 Using Dimensional Analysis on Units of Area**

A property in Italy is advertised at $45,000 for 6.8 hectares.

a. Find the area of the property in acres.
b. What is the price per acre?

**SOLUTION**

a. Using Table 9.4, we see that 1 acre = 0.4 hectare. To convert 6.8 hectares to acres, we use a unit fraction with acres in the numerator and hectares in the denominator.

\[
6.8 \text{ ha} = \frac{6.8 \text{ ha}}{1 \text{ acre}} \cdot \frac{1 \text{ acre}}{0.4 \text{ ha}} = 17 \text{ acres}
\]

The area of the property is approximately 17 acres.

b. The price per acre is the total price, $45,000, divided by the number of acres, 17.

\[
\text{price per acre} = \frac{$45,000}{17 \text{ acres}} = $32,941 / \text{acre}
\]

The price is approximately $32,941 per acre.

**CHECK POINT 4** A property in northern California is on the market at $415,000 for 1.8 acres.

a. Find the area of the property in hectares.
b. What is price per hectare?

**Measuring Volume**

A shoe box and a basketball are examples of three-dimensional figures. Volume refers to the amount of space occupied by such figures. In order to measure this space, we begin by selecting a cubic unit. Two such cubic units are shown in Figure 9.5.

**EXAMPLE 5 Measuring Volume**

What is the volume of the solid shown in Figure 9.6?

**SOLUTION**

We can determine the volume of the solid by counting the number of cubic units contained within the region. Because we have drawn a solid three-dimensional figure on a flat two-dimensional page, some of the small cubic units in the back, right are hidden. The figures below show how the cubic units are used to fill the inside of the solid.

Do these figures help you to see that there are 18 cubic units inside the solid? The volume of the solid is 18 cubic units.

**CHECK POINT 5** What is the volume of the region represented by the bottom row of blocks in Figure 9.6?

We have seen that there are 3 feet in a yard, but 9 square feet in a square yard. Neither of these relationships holds for cubic units. Figure 9.7 illustrates that there are 27 cubic feet in a cubic yard. Furthermore, there are 1728 cubic inches in a cubic foot.

**GREAT QUESTION!**

I'm having difficulty seeing the detail in Figure 9.7. Can you help me out?

Cutting numbers is helpful:

- 3 ft = 1 yd
- (3 yd)³ = 1 yd³

Conclusion: 27 ft³ = 1 yd³

- 12 in. = 1 ft
- (12 in.)³ = (1 ft)³

Conclusion: 1728 in.³ = 1 ft³

The measure of volume also includes the amount of fluid that a three-dimensional object can hold. This is often called the object's capacity. For example, we often refer to the capacity in gallons, of a gas tank. A cubic yard has a capacity of about 200 gallons and a cubic foot has a capacity of about 7.48 gallons.
Table 9.5 contains information about standard units of capacity in the English system.

<table>
<thead>
<tr>
<th>Volume in Cubic Units</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cubic yard</td>
<td>about 200 gallons</td>
</tr>
<tr>
<td>1 cubic foot</td>
<td>about 748 gallons</td>
</tr>
<tr>
<td>231 cubic inches</td>
<td>about 1 gallon</td>
</tr>
</tbody>
</table>

**EXAMPLE 6 Volume and Capacity in the English System**

A swimming pool has a volume of 22,500 cubic feet. How many gallons of water does the pool hold?

**SOLUTION**

We use the fact that 1 cubic foot has a capacity of about 748 gallons to set up our unit fraction:

\[
\frac{7.48 \text{ gal}}{1 \text{ ft}^3}
\]

We use this unit fraction to find the capacity of the 22,500 cubic feet.

\[
22,500 \text{ ft}^3 \times \frac{7.48 \text{ gal}}{1 \text{ ft}^3} = 22,500(7.48) \text{ gal} = 168,300 \text{ gal}
\]

The pool holds approximately 168,300 gallons of water.

**CHECK POINT 6** A pool has a volume of 10,000 cubic feet. How many gallons of water does the pool hold?

As we have come to expect, things are simpler when the metric system is used to measure capacity. The basic unit is the liter, symbolized by L. A liter is slightly larger than a quart.

\[
1 \text{ liter} \approx 1.0567 \text{ quarts}
\]

The standard metric prefixes are used to denote a multiple or part of a liter. Table 9.6 applies these prefixes to the liter.

**EXAMPLE 7 Volume and Capacity in the Metric System**

An aquarium has a volume of 36,000 cubic centimeters. How many liters of water does the aquarium hold?

**SOLUTION**

We use the fact that 1000 cubic centimeters corresponds to a capacity of 1 liter to set up our unit fraction:

\[
\frac{1 \text{ L}}{1000 \text{ cm}^3}
\]

We use this unit fraction to find the capacity of the 36,000 cubic centimeters.

\[
36,000 \text{ cm}^3 \times \frac{1 \text{ L}}{1000 \text{ cm}^3} = \frac{36,000 \text{ L}}{1000} = 36 \text{ L}
\]

The aquarium holds 36 liters of water.

**CHECK POINT 7** A fish pond has a volume of 220,000 cubic centimeters. How many liters of water does the pond hold?
Our next example involves measuring dosages of medicine in liquid form. We have seen that

\[ 1 \text{ cm}^3 = 1 \text{ mL} \]

Dosages of liquid medication are measured using cubic centimeters, or milliliters, as they are also called. In the United States, cc, rather than cm³, denotes cubic centimeters.

**EXAMPLE 8** Measuring Dosages of Medicine in Liquid Form

A physician orders 10 cc of the drug Lexapro (used to treat depression and anxiety) to be administered to a patient in liquid form.

**a.** How many milliliters of the drug should be administered?

**b.** How many fluid ounces of the drug should be administered?

**SOLUTION**

**a.** We use a relationship between volume and capacity in the metric system: 1 cubic centimeter (cc) = 1 milliliter (mL). Because 10 cubic centimeters (cc) of the drug is to be administered, this is equivalent to 10 milliliters (mL) of Lexapro.

**b.** We now need to convert 30 milliliters to fluid ounces. Table 9.8 on the previous page shows that 1 fluid ounce (fl oz) = 30 milliliters (mL). We use

\[ \frac{1 \text{ fl oz}}{30 \text{ mL}} \]

as our unit fraction.

\[ 10 \text{ mL} = 10 \text{ mL} \times \frac{1 \text{ fl oz}}{30 \text{ mL}} = \frac{10 \text{ fl oz}}{30} = 0.33 \text{ fl oz} \]

Approximately 0.33 fluid ounce (fl oz) of Lexapro should be administered.

**CHECK POINT 8** A physician orders 20 cc of the antibiotic Omnace to be administered every 12 hours.

**a.** How many milliliters of the drug should be administered?

**b.** How many fluid ounces of the drug should be administered? Round to two decimal places.
### Application Exercises

In Exercises 33–34, find the population density, to the nearest square mile, for each state. Which state has the greatest population density? How many more people per square mile inhabit the state with the greatest density than inhabit the state with the lesser density?

53. Illinois population: 12,969,257
   
   Area: 57,914 square miles
   
   Population density: 223.9 people per square mile

54. New York population: 19,465,197
   
   Area: 54,555 square miles
   
   Population density: 356.7 people per square mile

55. Rhode Island population: 1,051,302
   
   Area: 1,354 square miles
   
   Population density: 772.5 people per square mile

In Exercises 35–56, use the fact that 1 square mile = 640 acres to find the area of each national park to the nearest square mile.

56. Yosemite National Park (California): 761,268 acres
   
   Area: 1,217 square miles

### Practice Plus

The bar graph shows the resident population and the land area of the United States for selected years from 1800 through 2010. Use the information shown by the graph to solve Exercises 49–52.

![Bar Graph of Resident Population and Land Area](image)

#### U.S. Resident Population and Land Area

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
<th>Land Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1800</td>
<td>3,929,214</td>
<td>982,686</td>
</tr>
<tr>
<td>1900</td>
<td>88,221,568</td>
<td>982,686</td>
</tr>
<tr>
<td>1940</td>
<td>131,600,272</td>
<td>982,686</td>
</tr>
<tr>
<td>1980</td>
<td>229,491,557</td>
<td>982,686</td>
</tr>
<tr>
<td>2010</td>
<td>308,745,538</td>
<td>982,686</td>
</tr>
</tbody>
</table>

### Critical Thinking Exercises

#### Make Sense?

In Exercises 83–86, determine whether each statement makes sense or does not make sense, and explain your reasoning.

83. The capacity of my car is approximately 40 miles.
84. The population density of Montana is approximately 6.5 people per mile.
85. Yesterday I drank 2000 cm³ of water.
86. I read that one quart is approximately 3.0483L.
87. Singapore has the highest population density of any country: 66,690 people per 1000 hectares. How many people are there per square mile?
88. Nebraska has a population density of 2.38 people per square mile and a population of 1,342,641. What is the area of Nebraska? Round to the nearest square mile.
89. A high population density is a condition common to extremely poor and extremely rich locales. Explain why this is so.

#### Evaluation

19. A physician orders 3 teaspoons daily of indomethacin in three equally divided doses.
   - a. How many milliliters of the drug should be administered daily?
   - b. How many cc of the drug should be administered daily?
   - c. How many fluid ounces of the drug should be administered daily?
   - d. How many fluid ounces of the drug should the patient receive in each divided dose?

20. A physician orders 4 teaspoons daily of indomethacin in four equally divided doses.
   - a. How many milliliters of the drug should be administered daily?
   - b. How many cc of the drug should be administered daily?
   - c. How many fluid ounces of the drug should be administered daily?
   - d. How many fluid ounces of the drug should the patient receive in each divided dose?
Section 9.8

Concept and Vocabulary Check

1. 2) 12.5; 3) 3620; 4) 0.00372; 5) 21.69 kg; 6) 25.30 mm; 7) 55.16 cm; 8) 48.3 mm; 9) 0.12 m; 10) 472 mL

Section 9.9

Check Point Exercises

1. 2) 1; 3) 0.03; 4) 0.000372; 5) 21.69 kg; 6) 25.30 mm; 7) 55.16 cm; 8) 48.3 mm; 9) 0.12 m; 10) 472 mL

Section 9.10

Check Point Exercises

1. 2) 1; 3) 0.03; 4) 0.000372; 5) 21.69 kg; 6) 25.30 mm; 7) 55.16 cm; 8) 48.3 mm; 9) 0.12 m; 10) 472 mL