

DATE: 3A

**ESSENTIAL QUESTION(S):** How do you find a common denominator for rational expressions? How do you perform operations with rational expressions?

**REVIEW:**

Video for slant asymptote:



Step by step

- 1 Determine
  - a) top heavy  
- Slant Asymptote by division
  - b) bottom heavy  
HA = 0
  - c) equal to  
HA =  $\frac{\text{lead coef.}}{\text{lead coef.}}$
- 2 Find VA.  
denominator = 0
- 3 Find xint.  
numerator = 0
- 4 Find yint  
x = 0

**NOTES:**

**Review**

Graph the following improper rational functions.

1.  $f(x) = \frac{2x^2 - 7x + 6}{x - 1}$

1  $\begin{array}{r} 2 \overline{) 2 \ -7 \ 6} \\ \underline{2 \phantom{-} 0} \\ \phantom{2} -7 \ 6 \\ \phantom{2} \underline{-7 \phantom{0}} \\ \phantom{2} \phantom{-} 6 \phantom{0} \\ \phantom{2} \phantom{-} \underline{6 \phantom{0}} \\ \phantom{2} \phantom{-} \phantom{0} 0 \end{array}$       3  $2x^2 - 7x + 6 = 0$   
 $(2x - 3)(x - 2)$   
 $x = \frac{3}{2}, 2$

SA =  $2x - 5$

2  $x - 1 = 0$

VA  $x = 1$

2.  $f(x) = \frac{x^2 + 3x}{\frac{3x + 12}{3} \cdot x + 4}$

4  $\begin{array}{r} 4 \overline{) 1 \ 3 \ 0} \\ \underline{4 \phantom{0}} \\ \phantom{4} -1 \ 3 \ 0 \\ \phantom{4} \underline{-4 \phantom{0}} \\ \phantom{4} \phantom{-} 3 \ 0 \\ \phantom{4} \phantom{-} \underline{3 \ 0} \\ \phantom{4} \phantom{-} \phantom{0} 0 \end{array}$        $x^2 + 3x = 0$   
 $x(x + 3) = 0$   
 $x = 0, -3$

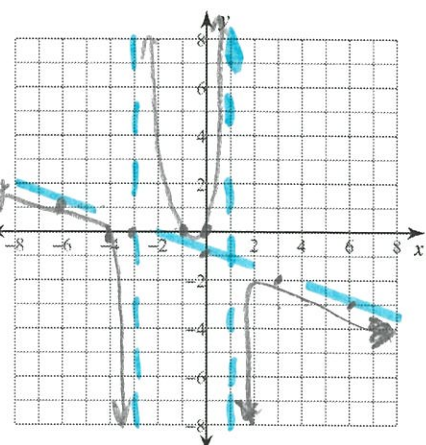
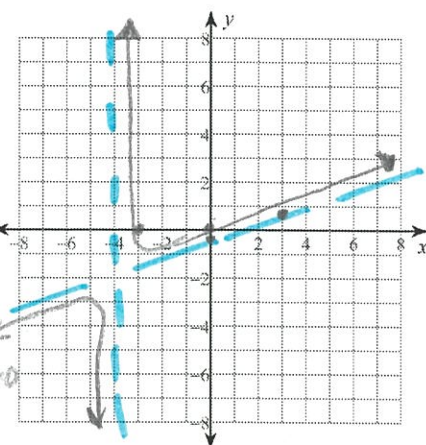
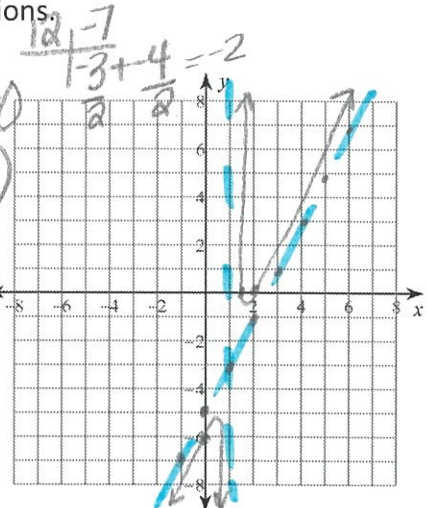
SA =  $\frac{1}{3}x - \frac{1}{3}$

VA = -4

3.  $f(x) = \frac{x^3 + 5x^2 + 4x}{-3x^2 - 6x + 9}$

SA =  $-\frac{1}{3}x - 1$

VA =  $-3(x^2 + 2x - 3)$   
 $-3(x - 1)(x + 3)$   
 $= 1, -3$   
 $x_{int} = x(x^2 + 5x + 4)$   
 $= x(x + 4)(x + 1)$   
 $= 0, -4, -1$



Video for adding and subtracting rational expressions:



- ① multiplied straight across made it one fraction
- ② factored form & cancelled.

### Multiplying Rational functions

Example using numbers	Example 1	Example 2
$\frac{3}{4} \cdot \frac{5}{6}$	$\frac{x(x-3)}{(x+1)} \cdot \frac{5}{x^2}$	$\frac{(x+1)(x-2)}{(x+2)} \cdot \frac{(x+5)}{(x-2)(x+2)}$
① $\frac{3 \cdot 5}{4 \cdot 6}$	$\frac{5x \cdot (x-3)}{x^2(x+1)}$	$\frac{(x+1)\cancel{(x-2)}(x+5)}{(x+2)\cancel{(x-2)}(x+2)}$
② $\frac{\cancel{3} \cdot 5}{2 \cdot 2 \cdot 2 \cdot \cancel{3}}$	$\frac{5 \cdot \cancel{x} (x-3)}{x \cdot \cancel{x} (x+1)}$	$\frac{(x+1)(x+5)}{(x+2)(x+2)}$
$\frac{5}{8}$	$\frac{5(x-3)}{x(x+1)}$ or $\frac{5x-15}{x(x+1)}$ leave in factored form	$\frac{(x+1)(x+5)}{(x+2)^2}$ or $\frac{x^2+6x+5}{(x+2)^2}$

### Dividing Rational functions

- ① Keep, Change, flip
- ② everything is factored then cancel
- ③ Combine any like terms

Example using numbers	Example 3	Example 4
$\frac{3}{4} \div \frac{5}{6}$	$\frac{(x-2)}{(x+2)} \div \frac{(x+5)}{x(x+2)}$	$\frac{(x+1)(x-6)}{(x+2)} \div \frac{(x+1)}{(x-3)(x+2)}$
① $\frac{3 \cdot 6}{4 \cdot 5}$	$\frac{(x-2)}{(x+2)} \cdot \frac{x(x+2)}{(x+5)}$	$\frac{\cancel{(x+1)}(x-6)}{(x+2)} \cdot \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+1)}}$
② $\frac{3 \cdot 6}{4 \cdot 5} = \frac{3 \cdot \cancel{2} \cdot 3}{2 \cdot 2 \cdot 5}$	$\frac{x(x-2)}{(x+5)}$	$(x-6)(x-3)$
③ $\frac{9}{10}$	$\frac{x(x-2)}{(x+5)}$ or $\frac{x^2-2x}{(x+5)}$	$(x-3)(x-6)$

Video for adding and subtracting rational expressions:



Add or sub fractions

①  $55 \frac{5}{7} + 63 \frac{9}{11}$

$$\frac{55+63}{77}$$

② multiply or combine like terms in the numerator

### Adding Rational expressions

Example using numbers	Example 5
$\frac{2}{3} + \frac{1}{7}$	① $\frac{3}{(x+7)} + \frac{4}{(x-4)}$
$\frac{2}{3} \left(\frac{7}{7}\right) + \frac{1}{7} \left(\frac{3}{3}\right)$	$\Rightarrow \frac{3}{(x+7)} \frac{(x-4)}{(x-4)} + \frac{4}{(x-4)} \frac{(x+7)}{(x+7)}$
$\frac{14}{21} + \frac{3}{21}$	$\frac{3(x-4) + 4(x+7)}{(x+7)(x-4)}$
$\frac{14+3}{21}$	② $\frac{3x-12 + 4x+28}{(x+7)(x-4)}$
$\frac{17}{21}$	$\frac{7x+16}{(x+7)(x-4)}$

### Example 6:

Add the following rational expressions.

$$\frac{2x-3}{x+3} + \frac{x+5}{x-2}$$

$$\frac{(2x-3)(x-2) + (x+3)(x+5)}{(x+3)(x-2)}$$

$$\frac{2x^2 - 4x - 3x + 6 + x^2 + 5x + 3x + 15}{(x+3)(x-2)}$$

$$\frac{3x^2 + 7x + 21}{(x+3)(x-2)}$$

### Subtracting Rational functions

if ...

$$(3x+1)(x-2) - (x+4)(x+5)$$

$$3x^2 - 5x - 2 - [x^2 + 4x + 5x + 20]$$

Example using numbers	Example 5
$\frac{7}{8} - \frac{3}{5}$	$\frac{3x+1}{(x+5)} - \frac{4}{(x-2)}$
$\frac{7}{8} \left(\frac{5}{5}\right) - \frac{3}{5} \left(\frac{8}{8}\right)$	$\frac{(3x+1)(x-2) - 4(x+5)}{(x+5)(x-2)}$
$\frac{35}{40} - \frac{24}{40}$	$\frac{3x^2 - 6x + x - 2 - 4x - 20}{(x+5)(x-2)}$
$\frac{35 - 24}{40}$	↓ ↓
$\frac{11}{40}$	$\frac{3x^2 - 9x - 22}{(x+5)(x-2)}$

#### Try on your own:

1. Henry simplified the following rational expressions. Only one of the three problems is correct. Determine which one he answered correctly. Then identify Henry's errors in the two that are incorrect and correct them.

a.  $\frac{5x}{(x-3)} + \frac{2}{(x-1)}$

$$\frac{5x(x-1)}{(x-3)(x-1)} + \frac{2(x-3)}{(x-3)(x-1)}$$

$$\frac{5x^2 - x + 2x - 3}{(x-3)(x-1)}$$

$$\frac{5x^2 + x - 3}{(x-3)(x-1)}$$

b.  $\frac{x}{(x+3)} - \frac{4(x+3)}{(x-1)}$

$$\frac{x}{1} - \frac{4}{(x-1)}$$

$$\frac{x(x-1)}{(x-1)} - \frac{4}{(x-1)}$$

$$\frac{x^2 - x - 4}{(x-1)}$$

c.  $\frac{(x+1)(x-2)}{(x+2)} \times \frac{(x+5)}{(x-2)(x+2)}$

$$\frac{(x+1)(x-2)(x+5)}{(x+2)(x-2)(x+2)}$$

$$\frac{(x+1)(x+5)}{(x+2)(x+2)}$$

$$\frac{x^2 + 6x + 5}{x^2 + 4x + 4}$$

2.  $\frac{2x+6}{(x+1)} - \frac{4}{(x+1)}$

3.  $\frac{2x}{(x+2)} + \frac{x-1}{(x-5)}$

4.  $\frac{x^2+6x+8}{x^2-5x+4} \cdot \frac{x^2+3x-4}{x^2+4x+4}$

5.  $\frac{2x}{(x^2-4)} + \frac{4}{(x+2)}$

6.  $\frac{4x+8}{5x-20} \div \frac{x^2-3x-10}{x^2-4x}$

7.  $\frac{x-10}{x-4} - \frac{x+2}{4-x}$

$$2) \frac{(2x+6)-4}{x+1} = \frac{2x+2}{x+1} = \frac{2(x+1)}{(x+1)} = \boxed{2}$$

$$3) \frac{2x}{(x+2)} + \frac{(x-1)}{(x-5)} = \frac{2x(x-5) + (x-1)(x+2)}{(x+2)(x-5)} = \frac{2x^2 - 10x + x^2 + 2x - 1x - 2}{(x+2)(x-5)}$$

$$= \frac{4x^2 - 9x - 2}{(x+2)(x-5)}$$

$$4) \frac{x^2+6x+8}{x^2-5x+4} \cdot \frac{x^2+3x-4}{x^2+4x+4} = \frac{\cancel{(x+2)}(x+4)(x+4)\cancel{(x-1)}}{(x-4)\cancel{(x-1)}\cancel{(x+2)}(x+2)} = \frac{(x+4)^2}{x-4}$$

$$5) \frac{2x}{(x^2-4)} + \frac{4}{(x+2)} = \frac{2x}{(x+2)(x-2)} + \frac{4}{(x+2)} = \frac{2x + 4(x-2)}{(x+2)(x-2)}$$

$$= \frac{2x + 4x - 8}{(x+2)(x-2)} = \frac{6x-8}{(x-2)(x+2)}$$

$$6) \frac{4x+8}{5x-20} \cdot \frac{x^2-4x}{x^2-3x-10}$$

$$\frac{4\cancel{(x+2)}}{5\cancel{(x-4)}} \cdot \frac{x\cancel{(x-4)}}{(x-5)\cancel{(x+2)}}$$

$$\frac{4x}{5(x-5)}$$

$$7) \frac{x-10}{x-4} - \frac{(x+2)}{4-x}$$

$$\frac{(x-10)}{(x-4)} + \frac{(x+2)}{(x-4)}$$

$$\frac{x-10+x+2}{x-4}$$

$$\frac{2x-8}{x-4} = \frac{2(x-4)}{(x-4)} = \boxed{2}$$

$$\begin{cases} 4-x \\ -x+4 \\ -1(x-4) \end{cases}$$

