

Find the standard form of the equation and state the key features

$$1) \quad 16x^2 + 9y^2 - 288x + 108y + 324 = 0$$

$$16x^2 - 288x + 9y^2 + 108y = -324$$

$$16(x^2 - 18x + \boxed{81}) + 9(y^2 + 12y + \boxed{36}) = -324 + 16\boxed{81} + 9\boxed{36}$$

$$\frac{16(x-9)^2}{1296} + \frac{9(y+6)^2}{1296} = \frac{1296}{1296}$$

$$\frac{(x-9)^2}{81} + \frac{(y+6)^2}{144} = 1$$

Center: $(9, -6)$

$$\frac{144-81}{\sqrt{63}} = 3\sqrt{7}$$

Foci: $(9, -6 + 3\sqrt{7})$ $(9, -6 - 3\sqrt{7})$

Vertices: $(18, -6)$ $(0, -6)$ $(9, 6)$ $(9, -18)$

1. A segment has endpoints with coordinates (2, 7) and (5, 1).
Find the length and midpoint of the segment.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{45} = 3\sqrt{5}$$

$$\begin{array}{l} (2-5)^2 + (7-1)^2 \\ (-3)^2 + (6)^2 \\ 9 + 36 \\ 45 \end{array}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left(\frac{7}{2}, 4 \right) \quad \frac{7}{2}, \frac{8}{2}$$

3. Find the center and radius of the circle with equation:

$$x^2 + y^2 - 6x + 10y - 1 = 0$$

$$x^2 - 6x + y^2 + 10y = 1$$

$$x^2 - 6x + \boxed{9} + y^2 + 10y + \boxed{25} = 1 + \boxed{9} + \boxed{25}$$

$$(x-3)^2 + (y+5)^2 = 35$$

Center: (3, -5) $r = \sqrt{35}$

4. Find the foci of the ellipse with the equation:

$$\frac{(x-1)^2}{16} + \frac{(y+1)^2}{25} = 1$$

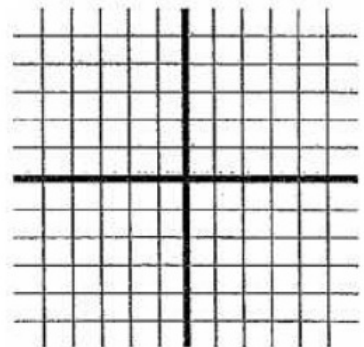
What type is this?

6. $5x^2 + 9y^2 = 45$

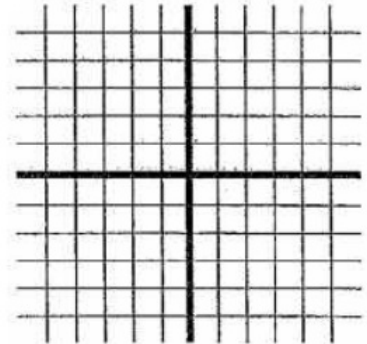
Ellipse
Horizontal

$$\frac{5x^2}{45} + \frac{9y^2}{45} = \frac{45}{45}$$

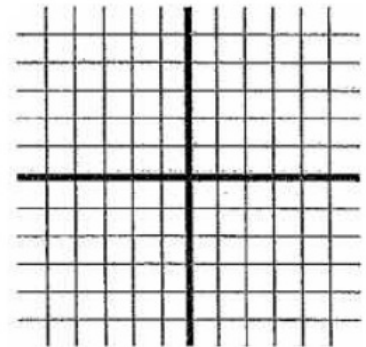
$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$



7. $x^2 + y^2 - 2x + 2y = 7$



10. $13x^2 + 13y^2 - 26x = 8$



13. Determine whether the quadrilateral $ABCD$ with vertices $A(-2, 2)$, $B(1, 3)$, $C(4, -1)$, and $D(1, -2)$ is a parallelogram.

14. Write the standard form of an equation of the circle that passes through the points $(-2, 16)$, $(-2, 0)$, and $(-32, 0)$. Then identify the center and radius.

$$\sqrt{?} = 9$$

$$\sqrt{?} = 3$$

Center: (a, b)

Distance from center to two points

$$(a+2)^2 + (b-16)^2 = (a+2)^2 + (b)^2$$

$$\begin{array}{r} a^2 + 4a + 4 + b^2 - 32b + 256 \\ -a^2 - 4a - 4 - b^2 \end{array} = \begin{array}{r} a^2 + 4a + 4 + b^2 \\ -a^2 - 4a - 4 - b^2 \end{array}$$

$$\begin{array}{r} -32b + 256 = 0 \\ -256 \quad -256 \\ \hline -32b = -256 \end{array}$$

$$b = 8$$

$$(a+2)^2 + \cancel{b^2} = (a+32)^2 + \cancel{b^2}$$

$$\begin{array}{r} (a+2)^2 = (a+32)^2 \\ a^2 + 4a + 4 = a^2 + 64a + 1024 \\ -a^2 - 4a - 1024 \quad -a^2 - 4a - 1024 \end{array}$$

$$-1020 = 60a$$

$$-17 = a$$

$$(x+17)^2 + (y-8)^2 = 289$$

$$(-17+2)^2 + 8^2$$

$$(-15)^2 + 64$$

$$225 + 64$$