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Study Guide

Graphs of Nonlinear Inequalities

Graphing an inequality in two variables identifies all ordered pairs that satisfy the inequality. The first step in graphing nonlinear inequalities is graphing the boundary.

Example 1   **Graph $y < \sqrt{x - 3} + 2$.**

The boundary of the inequality is the graph of $y = \sqrt{x - 3} + 2$. To graph the boundary curve, start with the parent graph $y = \sqrt{x}$. Analyze the boundary equation to determine how the boundary relates to the parent graph.

$$y = \sqrt{x - 3} + 2$$
$$\uparrow \quad \uparrow$$
move 3 units right   move 2 units up

Since the boundary is not included in the inequality, the graph is drawn as a dashed curve.

The inequality states that the y-values of the solution are less than the y-values on the graph of $y = \sqrt{x - 3} + 2$. Therefore, for a particular value of x, all of the points in the plane that lie below the curve have y-values less than $\sqrt{x - 3} + 2$. This portion of the graph should be shaded.

To verify numerically, test a point not on the boundary.

$$y < \sqrt{x - 3} + 2$$
$$0 \overset{?}{<} \sqrt{4 - 3} + 2 \quad \textit{Replace (x, y) with (4, 0).}$$
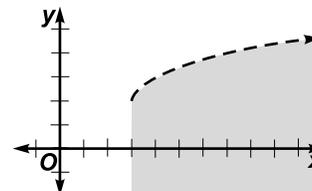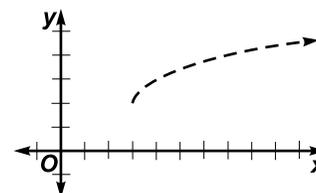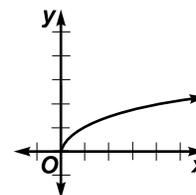$$0 < 3 \quad \checkmark \qquad \textit{True}$$

Since (4, 0) satisfies the inequality, the correct region is shaded.

Example 2   **Solve $\left| x - 3 \right| - 2 > 7$.**

Two cases must be solved. In one case, $x - 3$ is negative, and in the other, $x - 3$ is positive.

Case 1   If $a < 0$, then $\left| a \right| = -a$.   **Case 2**   If $a > 0$, then $\left| a \right| = a$.
$\quad\quad -(x - 3) - 2 > 7$ $\quad\quad x - 3 - 2 > 7$
$\quad\quad\quad -x + 3 - 2 > 7$ $\quad\quad\quad x - 5 > 7$
$\quad\quad\quad\quad -x > 6$ $\quad\quad\quad\quad x > 12$
$\quad\quad\quad\quad\quad x < -6$

The solution set is $\{x \mid x < -6 \text{ or } x > 12\}$.

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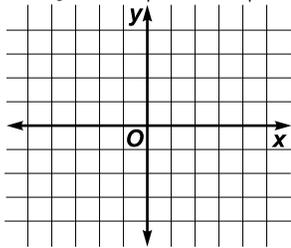
Practice

Graphs of Nonlinear Inequalities

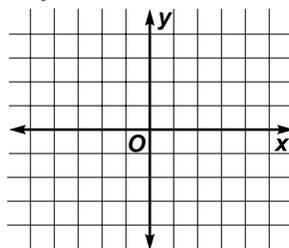*Determine whether the ordered pair is a solution for the given inequality.*
Write yes or no.

1. $y > (x + 2)^2 + 3, (-2, 6)$ **2.** $y < (x - 3)^3 + 2, (4, 5)$ **3.** $y \leq |2x - 4| - 1, (-4, 1)$
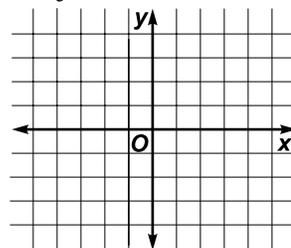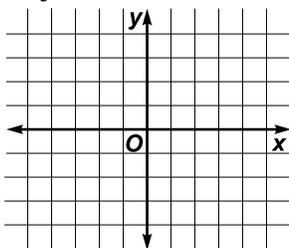
Graph each inequality.

4. $y \leq 2|x - 1|$

5. $y > 2(x - 1)^2$

6. $y < \sqrt{x - 2} + 1$

7. $y \geq (x + 3)^3$

Solve each inequality.

8. $|4x - 10| \leq 6$ **9.** $|x + 5| + 2 > 6$ **10.** $|2x - 2| - 1 < 7$

11. *Measurement*   Instructions for building a birdhouse warn that the platform, which ideally measures 14.75 cm², should not vary in size by more than 0.30 cm². If it does, the preconstructed roof for the birdhouse will not fit properly.

   a. Write an absolute value inequality that represents the range of possible sizes for the platform. Then solve for x to find the range.

   b. Dena cut a board 14.42 cm². Does the platform that Dena cut fit within the acceptable range?