

## Study Guide

### Inverse Functions and Relations

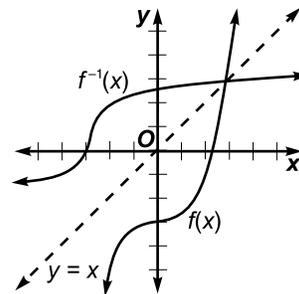
Two relations are inverse relations if and only if one relation contains the element  $(b, a)$  whenever the other relation contains the element  $(a, b)$ . If the inverse of the function  $f(x)$  is also a function, then the inverse is denoted by  $f^{-1}(x)$ .

**Example 1** Graph  $f(x) = \frac{1}{4}x^3 - 3$  and its inverse.

To graph the function, let  $y = f(x)$ . To graph  $f^{-1}(x)$ , interchange the  $x$ - and  $y$ -coordinates of the ordered pairs of the function.

| $f(x) = \frac{1}{4}x^3 - 3$ |       |
|-----------------------------|-------|
| $x$                         | $y$   |
| -3                          | -9.75 |
| -2                          | -5    |
| -1                          | -3.25 |
| 0                           | -3    |
| 1                           | -2.75 |
| 2                           | -1    |
| 3                           | 3.75  |

| $f^{-1}(x)$ |     |
|-------------|-----|
| $x$         | $y$ |
| -9.75       | -3  |
| -5          | -2  |
| -3.25       | -1  |
| -3          | 0   |
| -2.75       | 1   |
| -1          | 2   |
| 3.75        | 3   |

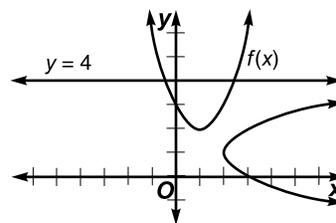


You can use the **horizontal line test** to determine if the inverse of a relation will be a function. If every horizontal line intersects the graph of the relation in at most one point, then the inverse of the relation is a function.

You can find the inverse of a relation algebraically. First, let  $y = f(x)$ . Then interchange  $x$  and  $y$ . Finally, solve the resulting equation for  $y$ .

**Example 2** Determine if the inverse of  $f(x) = (x - 1)^2 + 2$  is a function. Then find the inverse.

Since the line  $y = 4$  intersects the graph of  $f(x)$  at more than one point, the function fails the horizontal line test. Thus, the inverse of  $f(x)$  is not a function.



$$\begin{aligned}
 y &= (x - 1)^2 + 2 \\
 x &= (y - 1)^2 + 2 \\
 x - 2 &= (y - 1)^2 \\
 \pm\sqrt{x - 2} &= y - 1 \\
 y &= 1 \pm \sqrt{x - 2}
 \end{aligned}$$

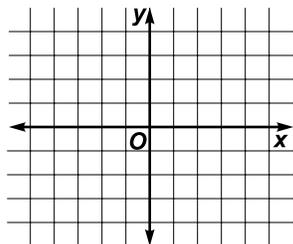
Let  $y = f(x)$ .  
 Interchange  $x$  and  $y$ .  
 Isolate the expression containing  $y$ .  
 Take the square root of each side.  
 Solve for  $y$ .

## Practice

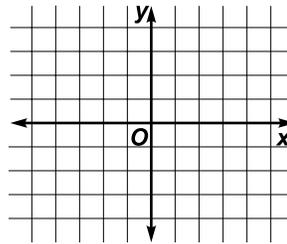
## Inverse Functions and Relations

Graph each function and its inverse.

1.  $f(x) = (x - 1)^3 + 1$



2.  $f(x) = 3|x| + 2$

Find the inverse of  $f(x)$ . Then state whether the inverse is also a function.

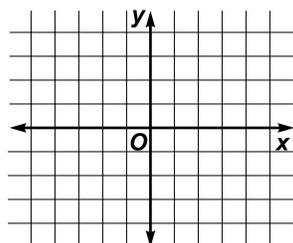
3.  $f(x) = -4x^2 + 1$

4.  $f(x) = \sqrt[3]{x - 1}$

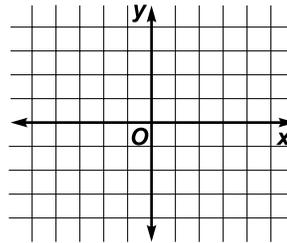
5.  $f(x) = \frac{4}{(x - 3)^2}$

Graph each equation using the graph of the given parent function.

6.  $y = -\sqrt{x + 3} - 1, p(x) = x^2$



7.  $y = 2 + \sqrt[5]{x + 2}, p(x) = x^5$



8. **Fire Fighting** Airplanes are often used to drop water on forest fires in an effort to stop the spread of the fire. The time  $t$  it takes the water to travel from height  $h$  to the ground can be derived from the equation  $h = \frac{1}{2}gt^2$  where  $g$  is the acceleration due to gravity (32 feet/second<sup>2</sup>).

- Write an equation that will give time as a function of height.
- Suppose a plane drops water from a height of 1024 feet. How many seconds will it take for the water to hit the ground?