NAME ______________________ DATE ______________ PERIOD _________

Study Guide

Graphs of Rational Functions

A **rational function** is a quotient of two polynomial functions.

The line $x = a$ is a **vertical asymptote** for a function $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$ from either the left or the right.

The line $y = b$ is a **horizontal asymptote** for a function $f(x)$ if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$.

A **slant asymptote** occurs when the degree of the numerator of a rational function is exactly one greater than that of the denominator.

Example 1 **Determine the asymptotes for the graph of $f(x) = \frac{2x - 1}{x + 3}$.**

Since $f(-3)$ is undefined, there may be a vertical asymptote at $x = -3$. To verify that $x = -3$ is a vertical asymptote, check to see that $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to -3$ from either the left or the right.

x	$f(x)$
−2.9	−68
−2.99	−698
−2.999	−6998
−2.9999	−69998

The values in the table confirm that $f(x) \to -\infty$ as $x \to -3$ from the right, so there is a vertical asymptote at $x = -3$.

One way to find the horizontal asymptote is to let $f(x) = y$ and solve for x in terms of y. Then find where the function is undefined for values of y.

$$y = \frac{2x - 1}{x + 3}$$
$$y(x + 3) = 2x - 1$$
$$xy + 3y = 2x - 1$$
$$xy - 2x = -3y - 1$$
$$x(y - 2) = -3y - 1$$
$$x = \frac{-3y - 1}{y - 2}$$

The rational expression $\frac{-3y - 1}{y - 2}$ is undefined for $y = 2$. Thus, the horizontal asymptote is the line $y = 2$.

Example 2 **Determine the slant asymptote for $f(x) = \frac{3x^2 - 2x + 2}{x - 1}$.**

First use division to rewrite the function.

$$\begin{array}{r} 3x + 1 \\ x - 1\overline{)3x^2 - 2x + 2} \\ \underline{3x^2 - 3x} \\ x + 2 \\ \underline{x - 1} \\ 3 \end{array} \quad \rightarrow \quad f(x) = 3x + 1 + \frac{3}{x - 1}$$

As $x \to \infty$, $\frac{3}{x - 1} \to 0$. Therefore, the graph of $f(x)$ will approach that of $y = 3x + 1$. This means that the line $y = 3x + 1$ is a slant asymptote for the graph of $f(x)$.

NAME ______________________________ DATE _______________ PERIOD ________

Practice

Graphs of Rational Functions

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

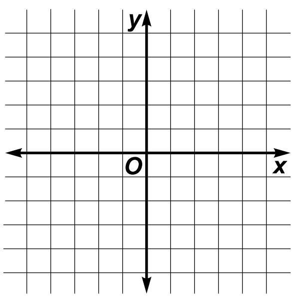
1. $f(x) = \frac{4}{x^2 + 1}$

2. $f(x) = \frac{2x + 1}{x + 1}$

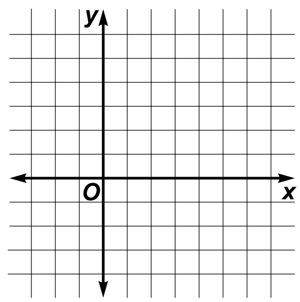
3. $g(x) = \frac{x + 3}{(x + 1)(x - 2)}$

Use the parent graph $f(x) = \frac{1}{x}$ to graph each equation. Describe the transformation(s) that have taken place. Identify the new locations of the asymptotes.

4. $y = \frac{3}{x + 1} - 2$



5. $y = -\frac{4}{x - 3} + 3$

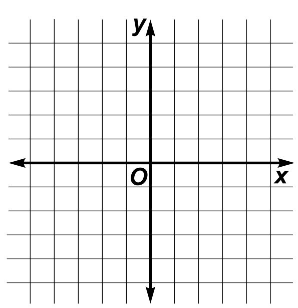


Determine the slant asymptotes of each equation.

6. $y = \frac{5x^2 - 10x + 1}{x - 2}$

7. $y = \frac{x^2 - x}{x + 1}$

8. Graph the function $y = \frac{x^2 + x - 6}{x + 1}$.



9. ***Physics*** The illumination I from a light source is given by the formula $I = \frac{k}{d^2}$, where k is a constant and d is distance. As the distance from the light source doubles, how does the illumination change?