

Study Guide

Polynomial Functions

The **degree** of a polynomial in one variable is the greatest exponent of its variable. The coefficient of the variable with the greatest exponent is called the **leading coefficient**. If a function $f(x)$ is defined by a polynomial in one variable, then it is a polynomial function. The values of x for which $f(x) = 0$ are called the **zeros** of the function. Zeros of the function are **roots** of the **polynomial equation** when $f(x) = 0$. A polynomial equation of degree n has exactly n roots in the set of complex numbers.

Example 1 State the degree and leading coefficient of the polynomial function $f(x) = 6x^5 + 8x^3 - 8x$. Then determine whether $\sqrt{\frac{2}{3}}$ is a zero of $f(x)$.

$6x^5 + 8x^3 - 8x$ has a degree of 5 and a leading coefficient of 6.
Evaluate the function for $x = \sqrt{\frac{2}{3}}$. That is, find $f\left(\sqrt{\frac{2}{3}}\right)$.

$$\begin{aligned} f\left(\sqrt{\frac{2}{3}}\right) &= 6\left(\sqrt{\frac{2}{3}}\right)^5 + 8\left(\sqrt{\frac{2}{3}}\right)^3 - 8\left(\sqrt{\frac{2}{3}}\right) & x &= \sqrt{\frac{2}{3}} \\ &= \frac{24}{9}\sqrt{\frac{2}{3}} + \frac{16}{3}\sqrt{\frac{2}{3}} - 8\sqrt{\frac{2}{3}} \\ &= 0 \end{aligned}$$

Since $f\left(\sqrt{\frac{2}{3}}\right) = 0$, $\sqrt{\frac{2}{3}}$ is a zero of $f(x) = 6x^5 + 8x^3 - 8x$.

Example 2 Write a polynomial equation of least degree with roots 0 , $\sqrt{2}i$, and $-\sqrt{2}i$.

The linear factors for the polynomial are $x - 0$, $x - \sqrt{2}i$, and $x + \sqrt{2}i$. Find the products of these factors.

$$\begin{aligned} (x - 0)(x - \sqrt{2}i)(x + \sqrt{2}i) &= 0 \\ x(x^2 - 2i^2) &= 0 \\ x(x^2 + 2) &= 0 & -2i^2 &= -2(-1) \text{ or } 2 \\ x^3 + 2x &= 0 \end{aligned}$$

Example 3 State the number of complex roots of the equation $3x^2 + 11x - 4 = 0$. Then find the roots.

The polynomial has a degree of 2, so there are two complex roots. Factor the equation to find the roots.

$$\begin{aligned} 3x^2 + 11x - 4 &= 0 \\ (3x - 1)(x + 4) &= 0 \end{aligned}$$

To find each root, set each factor equal to zero.

$$\begin{aligned} 3x - 1 &= 0 & x + 4 &= 0 \\ 3x &= 1 & x &= -4 \\ x &= \frac{1}{3} \end{aligned}$$

The roots are -4 and $\frac{1}{3}$.

Practice

Polynomial Functions

State the degree and leading coefficient of each polynomial.

1. $6a^4 + a^3 - 2a$

2. $3p^2 - 7p^5 - 2p^3 + 5$

Write a polynomial equation of least degree for each set of roots.

3. 3, -0.5, 1

4. 3, 3, 1, 1, -2

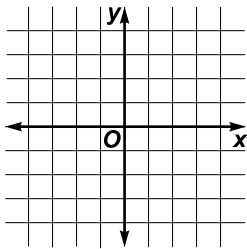
5. $\pm 2i$, 3, -3

6. -1, $3 \pm i$, $2 \pm 3i$

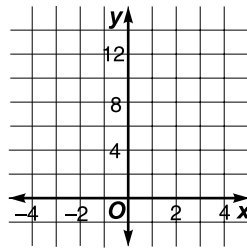
State the number of complex roots of each equation. Then find the roots and graph the related function.

7. $3x - 5 = 0$

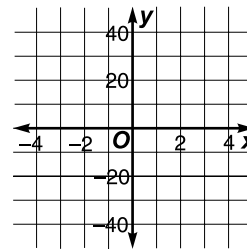
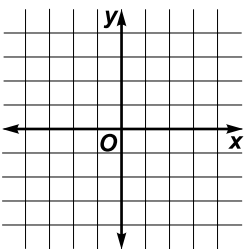
8. $x^2 + 4 = 0$



9. $c^2 + 2c + 1 = 0$



10. $x^3 + 2x^2 - 15x = 0$



11. **Real Estate** A developer wants to build homes on a rectangular plot of land 4 kilometers long and 3 kilometers wide. In this part of the city, regulations require a greenbelt of uniform width along two adjacent sides. The greenbelt must be 10 times the area of the development. Find the width of the greenbelt.