NAME ______________________ DATE ____________ PERIOD ________

Study Guide

Quadratic Equations

A quadratic equation is a polynomial equation with a degree of 2. Solving quadratic equations by graphing usually does not yield exact answers. Also, some quadratic expressions are not factorable. However, solutions can always be obtained by **completing the square.**

Example 1 **Solve $x^2 - 12x + 7 = 0$ by completing the square.**

$x^2 - 12x + 7 = 0$

$x^2 - 12x = -7$ *Subtract 7 from each side.*

$x^2 - 12x + 36 = -7 + 36$ *Complete the square by adding* $\left[\frac{1}{2}(-12)\right]^2$, *or 36, to each side.*

$(x - 6)^2 = 29$ *Factor the perfect square trinomial.*

$x - 6 = \pm\sqrt{29}$ *Take the square root of each side.*

$x = 6 \pm \sqrt{29}$ *Add 6 to each side.*

The roots of the equation are $6 \pm \sqrt{29}$.

Completing the square can be used to develop a general formula for solving any quadratic equation of the form $ax^2 + bx + c = 0$. This formula is called the **Quadratic Formula** and can be used to find the roots of any quadratic equation.

Quadratic Formula	If $ax^2 + bx + c = 0$ with $a \neq 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

In the Quadratic Formula, the radicand $b^2 - 4ac$ is called the **discriminant** of the equation. The discriminant tells the nature of the roots of a quadratic equation or the zeros of the related quadratic function.

Example 2 **Find the discriminant of $2x^2 - 3x = 7$ and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.**

Rewrite the equation using the standard form $ax^2 + bx + c = 0$.

$2x^2 - 3x - 7 = 0$ $\quad$ *$a = 2$, $b = -3$, and $c = -7$*

The value of the discriminant $b^2 - 4ac$ is $(-3)^2 - 4(2)(-7)$, or 65.

Since the value of the discriminant is greater than zero, there are two distinct real roots.

Now substitute the coefficients into the quadratic formula and solve.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{65}}{4}$$

The roots are $\frac{3 + \sqrt{65}}{4}$ and $\frac{3 - \sqrt{65}}{4}$.

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Practice

Quadratic Equations

Solve each equation by completing the square.

1. $x^2 - 5x - \frac{11}{4} = 0$

2. $-4x^2 - 11x = 7$

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

3. $x^2 + x - 6 = 0$

4. $4x^2 - 4x - 15 = 0$

5. $9x^2 - 12x + 4 = 0$

6. $3x^2 + 2x + 5 = 0$

Solve each equation.

7. $2x^2 + 5x - 12 = 0$

8. $5x^2 - 14x + 11 = 0$

9. ***Architecture*** The ancient Greek mathematicians thought that the most pleasing geometric forms, such as the ratio of the height to the width of a doorway, were created using the *golden section*. However, they were surprised to learn that the golden section is not a rational number. One way of expressing the golden section is by using a line segment. In the line segment shown, $\frac{AB}{AC} = \frac{AC}{CB}$. If $AC = 1$ unit, find the ratio $\frac{AB}{AC}$.