Study Guide

The Rational Root Theorem

The **Rational Root Theorem** provides a means of determining possible rational roots of an equation. **Descartes' Rule of Signs** can be used to determine the possible number of positive real zeros and the possible number of negative real zeros.

Rational Root Theorem	Let $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ represent a polynomial equation of degree n with integral coefficients. If a rational number $\frac{p}{q}$, where p and q have no common factors, is a root of the equation, then p is a factor of a_n and q is a factor of a_0.

Example 1 **List the possible rational roots of $x^3 - 5x^2 - 17x - 6 = 0$. Then determine the rational roots.**

p is a factor of 6 and q is a factor of 1
possible values of p: $\pm 1, \pm 2, \pm 3, \pm 6$
possible values of q: ± 1
possible rational roots, $\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 6$

Test the possible roots using synthetic division.

r	1	−5	−17	−6
1	1	−4	−21	−27
−1	1	−6	−11	5
2	1	−3	−23	−52
−2	1	−7	−3	0
3	1	−2	−23	−75
−3	1	−8	7	−27
6	1	1	−11	−72
−6	1	−11	49	−300

← *There is a root at $x = -2$.*
The depressed polynomial is $x^2 - 7x - 3$. You can use the Quadratic Formula to find the two irrational roots.

Example 2 **Find the number of possible positive real zeros and the number of possible negative real zeros for $f(x) = 4x^4 - 13x^3 - 21x^2 + 38x - 8$.**

According to Descartes' Rule of Signs, the number of positive real zeros is the same as the number of sign changes of the coefficients of the terms in descending order or is less than this by an even number. Count the sign changes.

$$f(x) = 4x^4 - 13x^3 - 21x^2 + 38x - 8$$
$$4 \quad -13 \quad -21 \quad 38 \quad -8$$

There are three changes. So, there are 3 or 1 positive real zeros.

The number of negative real zeros is the same as the number of sign changes of the coefficients of the terms of $f(-x)$, or less than this number by an even number.

$$f(-x) = 4(-x)^4 - 13(-x)^3 - 21(-x)^2 + 38(-x) - 8$$
$$4 \quad 13 \quad -21 \quad -38 \quad -8$$

There is one change. So, there is 1 negative real zero

NAME ______________________ DATE ____________ PERIOD _______

4-4

Practice

The Rational Root Theorem

List the possible rational roots of each equation. Then determine the rational roots.

1. $x^3 - x^2 - 8x + 12 = 0$

2. $2x^3 - 3x^2 - 2x + 3 = 0$

3. $36x^4 - 13x^2 + 1 = 0$

4. $x^3 + 3x^2 - 6x - 8 = 0$

5. $x^4 - 3x^3 - 11x^2 + 3x + 10 = 0$

6. $x^4 + x^2 - 2 = 0$

7. $3x^3 + x^2 - 8x + 6 = 0$

8. $x^3 + 4x^2 - 2x + 15 = 0$

Find the number of possible positive real zeros and the number of possible negative real zeros. Then determine the rational zeros.

9. $f(x) = x^3 - 2x^2 - 19x + 20$

10. $f(x) = x^4 + x^3 - 7x^2 - x + 6$

11. ***Driving*** An automobile moving at 12 meters per second on level ground begins to decelerate at a rate of -1.6 meters per second squared. The formula for the distance an object has traveled is $d(t) = v_0 t + \frac{1}{2}at^2$, where v_0 is the initial velocity and a is the acceleration. For what value(s) of t does $d(t) = 40$ meters?