NAME ______________________ DATE ____________ PERIOD ________

Study Guide

Rational Equations and Partial Fractions

A **rational equation** consists of one or more rational expressions. One way to solve a rational equation is to multiply each side of the equation by the least common denominator (LCD). Any possible solution that results in a zero in the denominator must be excluded from your list of solutions. In order to find the LCD, it is sometimes necessary to factor the denominators. If a denominator can be factored, the expression can be rewritten as the sum of **partial fractions.**

Example 1 **Solve $\frac{x+1}{3(x-2)} = \frac{5x}{6} + \frac{1}{x-2}$.**

$6(x-2)\left[\frac{x+1}{3(x-2)}\right] = 6(x-2)\left(\frac{5x}{6} + \frac{1}{x-2}\right)$ *Multiply each side by the LCD, 6(x − 2).*

$2(x+1) = (x-2)(5x) + 6(1)$

$2x + 2 = 5x^2 - 10x + 6$ *Simplify.*

$5x^2 - 12x + 4 = 0$ *Write in standard form.*

$(5x-2)(x-2) = 0$ *Factor.*

$5x - 2 = 0$ $\quad$ $x - 2 = 0$

$x = \frac{2}{5}$ $\quad$ $x = 2$

Since x cannot equal 2 because a zero denominator results, the only solution is $\frac{2}{5}$.

Example 2 **Decompose $\frac{2x-1}{x^2+2x-3}$ into partial fractions.**

Factor the denominator and express the factored form as the sum of two fractions using A and B as numerators and the factors as denominators.

$x^2 + 2x - 3 = (x-1)(x+3)$

$\frac{2x-1}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3}$

$2x - 1 = A(x+3) + B(x-1)$

Let $x = 1$. $\quad$ Let $x = -3$.

$2(1) - 1 = A(1+3)$ $\quad$ $2(-3) - 1 = B(-3-1)$

$1 = 4A$ $\quad$ $-7 = -4B$

$A = \frac{1}{4}$ $\quad$ $B = \frac{7}{4}$

$\frac{2x-1}{x^2+2x-3} = \frac{\frac{1}{4}}{x-1} + \frac{\frac{7}{4}}{x+3}$ or $\frac{1}{4(x-1)} + \frac{7}{4(x+3)}$

Example 3 **Solve $\frac{1}{2t} + \frac{3}{4t} > 1$.**

Rewrite the inequality as the related function $f(t) = \frac{1}{2t} + \frac{3}{4t} - 1$.

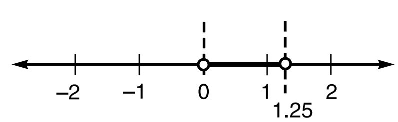
Find the zeros of this function.

$4t\left(\frac{1}{2t}\right) + 4t\left(\frac{3}{4t}\right) - 4t(1) = 4t(0)$

$5 - 4t = 0$

$t = 1.25$

The zero is 1.25. The excluded value is 0. On a number line, mark these values with vertical dashed lines. Testing each interval shows the solution set to be $0 < t < 1.25$.



NAME ______________________________ DATE ________________ PERIOD _________

Practice

Rational Equations and Partial Fractions

Solve each equation.

1. $\frac{15}{m} - m + 8 = 10$

2. $\frac{4}{b-3} + \frac{3}{b} = \frac{-2b}{b-3}$

3. $\frac{1}{2n} + \frac{6n-9}{3n} = \frac{2}{n}$

4. $t - \frac{4}{t} = 3$

5. $\frac{3a}{2a+1} - \frac{4}{2a-1} = 1$

6. $\frac{2p}{p+1} + \frac{3}{p-1} = \frac{15-p}{p^2-1}$

Decompose each expression into partial fractions.

7. $\frac{-3x-29}{x^2-4x-21}$

8. $\frac{11x-7}{2x^2-3x-2}$

Solve each inequality.

9. $\frac{6}{t} + 3 > \frac{2}{t}$

10. $\frac{2n+1}{3n+1} \leq \frac{n-1}{3n+1}$

11. $1 + \frac{3y}{1-y} > 2$

12. $\frac{2x}{4} - \frac{5x+1}{3} > 3$

13. ***Commuting*** Rosea drives her car 30 kilometers to the train station, where she boards a train to complete her trip. The total trip is 120 kilometers. The average speed of the train is 20 kilometers per hour faster than that of the car. At what speed must she drive her car if the total time for the trip is less than 2.5 hours?