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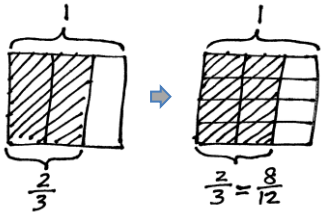
Grade 5 • Module 3

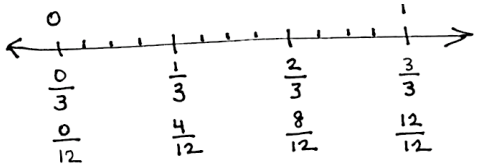
Addition and Subtraction of Fractions

OVERVIEW

In Module 3, students' understanding of addition and subtraction of fractions extends from earlier work with fraction equivalence and decimals. This module marks a significant shift away from the elementary grades' centrality of base ten units to the study and use of the full set of fractional units from Grade 5 forward, especially as applied to algebra.

In Topic A, students revisit the foundational Grade 4 standards addressing equivalence. When equivalent, fractions represent the same amount of area of a rectangle and the same point on the number line. These equivalencies can also be represented symbolically.





$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Furthermore, equivalence is evidenced when adding fractions with the same denominator. The sum may be decomposed into parts (or recomposed into an equal sum). An example is shown as follows:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

$$\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$$

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

This also carries forward work with decimal place value from Modules 1 and 2, confirming that like units can be composed and decomposed.

5 tenths + 7 tenths = 12 tenths = 1 and 2 tenths

5 eighths + 7 eighths = 12 eighths = 1 and 4 eighths

In Topic B, students move forward to see that fraction addition and subtraction are analogous to whole number addition and subtraction. Students add and subtract fractions with unlike denominators (**5.NF.1**) by replacing different fractional units with an equivalent fraction or like unit.

1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

$$\frac{1}{4}+\frac{2}{3}=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$

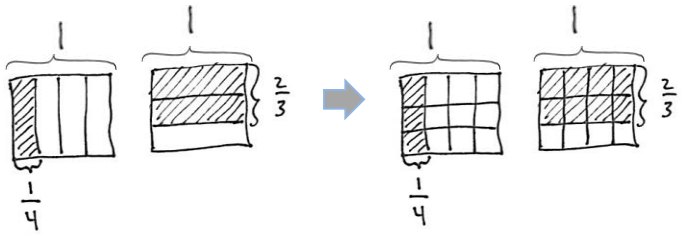
This is not a new concept, but certainly a new level of complexity. Students have added equivalent or like units since kindergarten, adding frogs to frogs, ones to ones, tens to tens, etc.

1 boy + 2 girls = 1 child + 2 children = 3 children

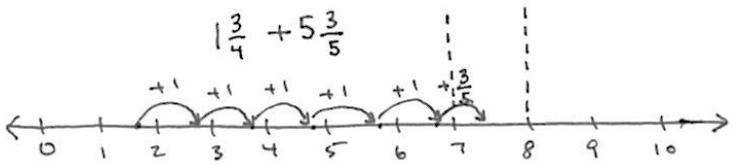
1 liter – 375 mL = 1,000 mL – 375 mL = 625 mL

Throughout the module, a concrete to pictorial to abstract approach is used to convey this simple concept. Topic A uses paper strips and number line diagrams to clearly show equivalence. After a brief concrete experience with folding paper, Topic B primarily uses the rectangular fractional model because it is useful for creating smaller like units by means of partitioning (e.g., thirds and fourths are changed to twelfths to create equivalent fractions as in the diagram below). In Topic C, students move away from the pictorial altogether as they are empowered to write equations clarified by the model.

$$\frac{1}{4}+\frac{2}{3}=\left(\frac{1\times 3}{4\times 3}\right)+\left(\frac{2\times 4}{3\times 4}\right)=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$



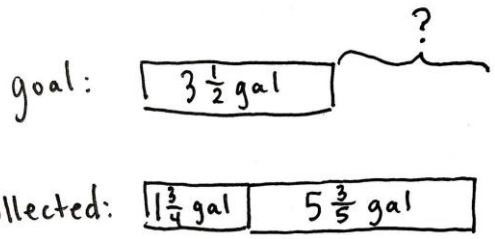
Topic C also uses the number line when adding and subtracting fractions greater than or equal to 1 so that students begin to see and manipulate fractions in relation to larger whole numbers and to each other. The number line allows students to pictorially represent larger whole numbers. For example, "Between which two whole numbers does the sum of $1\frac{3}{4}$ and $5\frac{3}{5}$ lie?"



____ < $1\frac{3}{4}$ + $5\frac{3}{5}$ < ____

This leads to an understanding of and skill with solving more complex problems, which are often embedded within multi-step word problems:

> *Cristina and Matt's goal is to collect a total of* $3\frac{1}{2}$ *gallons of sap from the maple trees. Cristina collected* $1\frac{3}{4}$ *gallons. Matt collected* $5\frac{3}{5}$ *gallons. By how much did they beat their goal?*



$$1\frac{3}{4}+5\frac{3}{5}-3\frac{1}{2}=3+\left(\frac{3\times 5}{4\times 5}\right)+\left(\frac{3\times 4}{5\times 4}\right)-\left(\frac{1\times 10}{2\times 10}\right)$$

$$=3+\frac{15}{20}+\frac{12}{20}-\frac{10}{20}=3\frac{17}{20}$$

Cristina and Matt beat their goal by $3\frac{17}{20}$ gallons.

Word problems are a part of every lesson. Students are encouraged to draw tape diagrams, which encourage them to recognize part–whole relationships with fractions that they have seen with whole numbers since Grade 1.

In Topic D, students strategize to solve multi-term problems and more intensely assess the reasonableness of their solutions to equations and word problems with fractional units (**5.NF.2**).

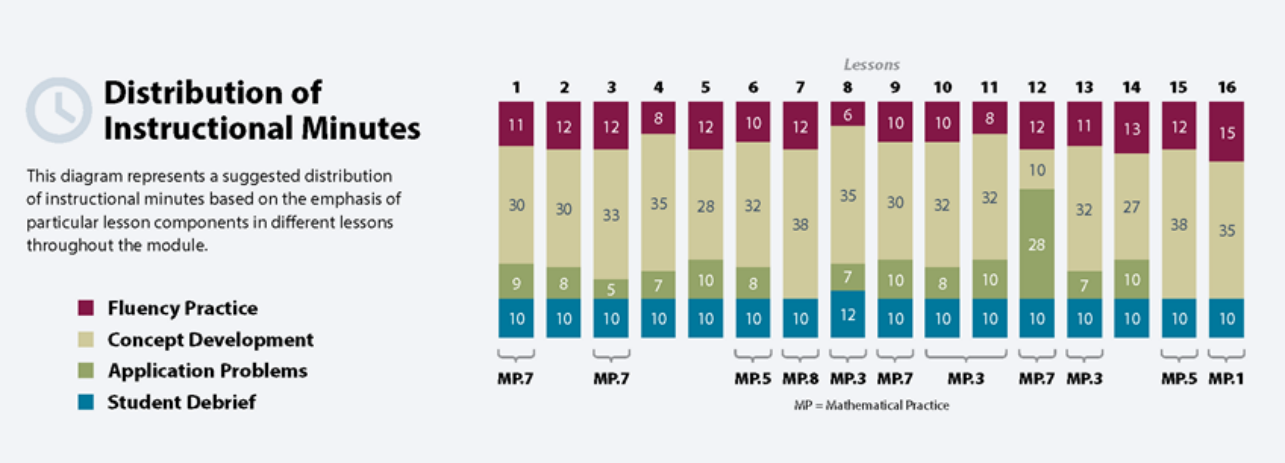
> *"I know my answer makes sense because the total amount of sap they collected is about 7 and a half gallons. Then, when we subtract 3 gallons, that is about 4 and a half. Then, 1 half less than that is about 4.* $3\frac{17}{20}$ *is just a little less than 4."*

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications and omissions. Omit Lesson 2 as it addresses a Grade 4 standard. In Lesson 3, omit the paper folding exercise, and consider it a remediation tool. Omit the Sprint in Lesson 12, and replace it with simple reasoning about fractions on the number line, such as "Is $\frac{3}{4}$ greater than or less than $\frac{1}{2}$? $\frac{3}{5}$? $\frac{3}{7}$?" In Lesson 15, choose two or three problems, and omit the others. Use the omitted problems as Application Problems in future lessons. Consider omitting Lesson 16 and using it in a center for early finishers, or have advanced students work the problems and present their solutions in a video or interactive demonstration. Consider asking the following questions to students, "Have you ever thought about what the whole would look like if this paper were one-half? What if it were one-third? What if this is three-fourths of the whole? What would the whole look like then?"

Note: In the first year of implementation, beginning in Lesson 5, be sure to include the fluency activities requiring students to subtract fractions less than one from a whole number (e.g., $4-\frac{5}{8}$) in order to prepare students to subtract larger mixed numbers in Topics B and C. Model these fluency activities on the number line and with a tape diagram.



Focus Grade Level Standards

Use equivalent fractions as a strategy to add and subtract fractions.[1]

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Foundational Standards

4.NF.1 Explain why a fraction *a/b* is equivalent to a fraction *(n x a)/(n x b)* by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.3 Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

[1]Examples in this module also include tenths and hundredths in fraction and decimal form.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of problems when they use number lines, tape diagrams, and fraction models to conceptualize and solve fraction addition and subtraction problems. Students also check their work and monitor their progress, assessing their approach and its validity within the given context and altering their method when necessary.

MP.3 **Construct viable arguments and critique the reasoning of others.** As students add and subtract with fractions and mixed numbers, they make choices and reason about which like unit to choose and draw conclusions about what makes some problems simpler than others. Students analyze multiple solution strategies for given problems and draw conclusions about which method is most efficient in each case. Students also critique the reasoning of others and construct viable arguments during this analysis. They also use their understanding of fractions to assess the reasonableness of sums and differences and use these assumptions to justify their conclusions to others.

MP.5 **Use appropriate tools strategically**. Students use mental computation and estimation strategies to assess the reasonableness of their answers. They decide which pictorial model to draw and label and reason about its size relative to the context of the problem. Students decide on the appropriateness of using special strategies when adding and subtracting mixed numbers.

MP.7 **Look for and make use of structure.** Students discern patterns and structures as they draw fraction models and reason about the number of units represented, the size or length of those units, and the name of the fraction that each model represents. They identify patterns in sums and differences when the same fraction is added to or taken from a variety of numbers and use this understanding to generate predictions about the sums and differences.

MP.8 **Look for and express regularity in repeated reasoning.** Students express regularity in repeated reasoning when they look for and use whole number general methods to add and subtract fractions. Adding and subtracting fractions requires finding like units just as it does with whole numbers, such as when adding centimeters and meters.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.NF.1 4.NF.3c 4.NF.3d	A	**Equivalent Fractions** Lesson 1: Make equivalent fractions with the number line, the area model, and numbers. Lesson 2: Make equivalent fractions with sums of fractions with like denominators.	2
5.NF.1 **5.NF.2**	B	**Making Like Units Pictorially** Lesson 3: Add fractions with unlike units using the strategy of creating equivalent fractions. Lesson 4: Add fractions with sums between 1 and 2. Lesson 5: Subtract fractions with unlike units using the strategy of creating equivalent fractions. Lesson 6: Subtract fractions from numbers between 1 and 2. Lesson 7: Solve two-step word problems.	5
		Mid-Module Assessment: Topics A–B (assessment ½ day, return ½ day, remediation or further applications 2 days)	3
5.NF.1 **5.NF.2**	C	**Making Like Units Numerically** Lesson 8: Add fractions to and subtract fractions from whole numbers using equivalence and the number line as strategies. Lesson 9: Add fractions making like units numerically. Lesson 10: Add fractions with sums greater than 2. Lesson 11: Subtract fractions making like units numerically. Lesson 12: Subtract fractions greater than or equal to 1.	5
5.NF.1 **5.NF.2**	D	**Further Applications** Lesson 13: Use fraction benchmark numbers to assess reasonableness of addition and subtraction equations. Lesson 14: Strategize to solve multi-term problems. Lesson 15: Solve multi-step word problems; assess reasonableness of solutions using benchmark numbers. Lesson 16: Explore part-to-whole relationships.	4

Standards	Topics and Objectives	Days
	End-of-Module Assessment: Topics A–D (assessment ½ day, return ½ day, remediation or further applications 2 days)	3
Total Number of Instructional Days		**22**

Terminology

New or Recently Introduced Terms

- Benchmark fraction (e.g., $\frac{1}{2}$ is a benchmark fraction when comparing $\frac{1}{3}$ and $\frac{3}{5}$)
- Like denominators (e.g., $\frac{1}{8}$ and $\frac{5}{8}$)
- Unlike denominators (e.g., $\frac{1}{8}$ and $\frac{1}{7}$)

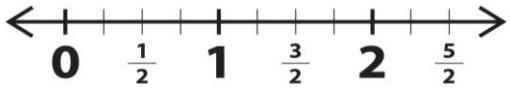
Familiar Terms and Symbols[2]

- Between (e.g., $\frac{1}{2}$ is between $\frac{1}{3}$ and $\frac{3}{5}$)
- Denominator (denotes the fractional unit: fifths in 3 fifths, which is abbreviated as the 5 in $\frac{3}{5}$)
- Equivalent fraction (e.g., $\frac{3}{5} = \frac{6}{10}$)
- Fraction (e.g., 3 fifths or $\frac{3}{5}$)
- Fraction greater than or equal to 1 (e.g., $\frac{7}{3}$, $3\frac{1}{2}$, an abbreviation for $3 + \frac{1}{2}$)
- Fraction written in the largest possible unit (e.g., $\frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ or 1 three out of 2 threes = $\frac{1}{2}$)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in $\frac{3}{5}$)
- Hundredth ($\frac{1}{100}$ or 0.01)
- Kilometer, meter, centimeter, liter, milliliter, kilogram, gram, mile, yard, foot, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- *More than halfway* and *less than halfway*
- Number sentence (e.g., Three plus seven equals ten. Usually written as *3 + 7 = 10*.)
- Numerator (denotes the count of fractional units: 3 in 3 fifths or 3 in $\frac{3}{5}$)
- *One tenth of* (e.g., $\frac{1}{10} \times 250$)
- Tenth ($\frac{1}{10}$ or 0.1)
- Whole unit (e.g., any unit that is partitioned into smaller, equally sized fractional units)
- <, >, =

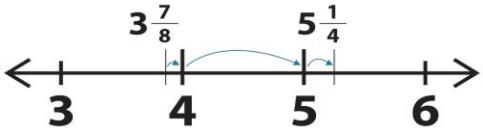
[2]These are terms and symbols students have seen previously.

Suggested Tools and Representations

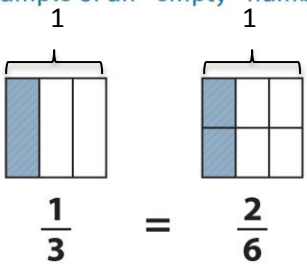
- Fraction strips
- Number line (a variety of templates)
- Paper strips (for modeling equivalence)
- Rectangular fraction model
- Tape diagrams



Example of a number line



Example of an "empty" number line



Example of a rectangular fraction model

Scaffolds[3]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	5.NF.1 5.NF.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	5.NF.1 5.NF.2

[3]Students with disabilities may require Braille, large-print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.