Task 3.4 – Simple Is as Simple Does

We have often seen that multiple equations can represent the same function. This is because these equations are really just different forms of the same equation. For example...

$$y = 3x + 4 \quad \text{and} \quad y = 3(x - 2) + 10$$

...are just slope-intercept and point-slope form equations of the same line. The same idea can work for numbers, like how $\frac{12}{3}$, $\frac{52}{13}$, and $\frac{200}{50}$ are all just alternative ways of writing the number 4. Today we will look for a similar relationship in radicals.

1. Use a calculator to approximate the value of each of the following radicals. Round your answers to the first decimal place:

$4\sqrt{5} =$ 8.9 $\qquad 2\sqrt{20} =$ 8.9 $\qquad \sqrt{80} =$ 8.9

A quick check with a calculator shows us that even though the numbers looked different, they are in fact the same. Today's goal is to figure out exactly why these values are the same, and then to extend that idea to include more complicated radicals ("radical" is a fancy word for square roots, cube roots, etc.).

2. See if you can simplify each of the following expressions in any way:

$2^3 * 3^3 =$ 216 6^3 $\qquad 5^4 * 11^4 =$ 55^4 $\qquad a^x * b^x =$ $(ab)^x$ $\qquad 4^{1/2} * 5^{1/2} =$ $20^{\frac{1}{2}}$

Rational exponents are equivalent to radicals, so a rule that works in exponential form, like $4^{1/2} * 5^{1/2} = 20^{1/2}$, must also work in radical form, like $\sqrt{4} \cdot \sqrt{5} = \sqrt{4 \cdot 5}$. This creates a brand new rule for dealing with radicals:

$$\boxed{\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}}$$

Using this same property, we can demonstrate that the above answers must be equal.

3. Rewrite 28 as the product of two factors, one of which is a square number. Then use the above property to simplify $\sqrt{28}$:

$\sqrt{28} = \sqrt{4 \cdot 7}$
$= \sqrt{4} \cdot \sqrt{7}$
$= 2\sqrt{7}$

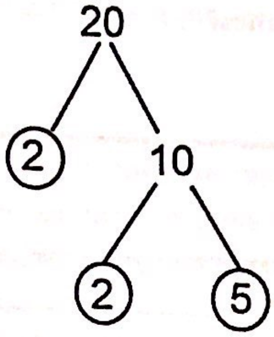
$4 \cdot 7$
$2 \cdot 14$
$1 \cdot 28$

4. Try simplifying each of the following square roots the same way we simplified $\sqrt{28}$

$\sqrt{18} = \sqrt{9 \cdot 2}$, $\sqrt{9}\sqrt{2}$, $3\sqrt{2}$ $\qquad \sqrt{75} = \sqrt{25 \cdot 3}$, $\sqrt{25}\sqrt{3}$, $5\sqrt{3}$ $\qquad \sqrt{72} = \sqrt{36 \cdot 2}$, $\sqrt{36}\sqrt{2}$, $6\sqrt{2}$ $\qquad \sqrt{24} = \sqrt{4 \cdot 6}$, $\sqrt{4}\sqrt{6}$, $2\sqrt{6}$

$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

Another way you might simplify radicals is by using prime factorization. For example, if I wanted to calculate $\sqrt{20}$, I would first build a factor tree and find the prime factorization.



Then I would rewrite and solve my problem using that prime factorization:

$\sqrt{2 \cdot 2 \cdot 5}$

$2\sqrt{5}$

$$\sqrt{20} = \sqrt{2^2 \cdot 5}$$
$$= \sqrt{2^2} \cdot \sqrt{5}$$
$$= 2\sqrt{5}$$

5. Try simplifying the same square roots using this new method. You might find the last one a little trickier:

$\sqrt[2]{18} =$ 18, 2, 9, 3, 3 — $\sqrt{2 \cdot 3 \cdot 3}$ — $3\sqrt{2}$

$\sqrt{75} =$ 15, 5, 3, 5 — $\sqrt{3 \cdot 5 \cdot 5}$ — $5\sqrt{3}$

$\sqrt{72} =$ 36, 2, 6, 6, 3, 2, 3, 2 — $\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$ — $2 \cdot 3\sqrt{2} = 6\sqrt{2}$

$\sqrt{24} =$ 2, 12, 2, 6, 2, 3 — $\sqrt{2 \cdot 2 \cdot 2 \cdot 3}$ — $2\sqrt{6}$

6. By using the prime factorization, we are able to simplify more complex radicals. Try simplifying each of the following. Remember, the inverse of a 3rd root is a third power, the inverse of a 4th root is a 4th power, etc.

$\sqrt[3]{24} =$ 12, 2, 4, 3, 2, 2 — $\sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$ — $2\sqrt[3]{3}$

$\sqrt[4]{80} =$ $2\sqrt[4]{5}$ — 8, 10, 2, 4, 5, 2, 2, 2 — $\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}$

$\sqrt[3]{162} =$ 2, 81, 3, 27, 3, 9, 3, 3 — $\sqrt{2 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$ — $3\sqrt{2 \cdot 3}$ — $3\sqrt[3]{6}$